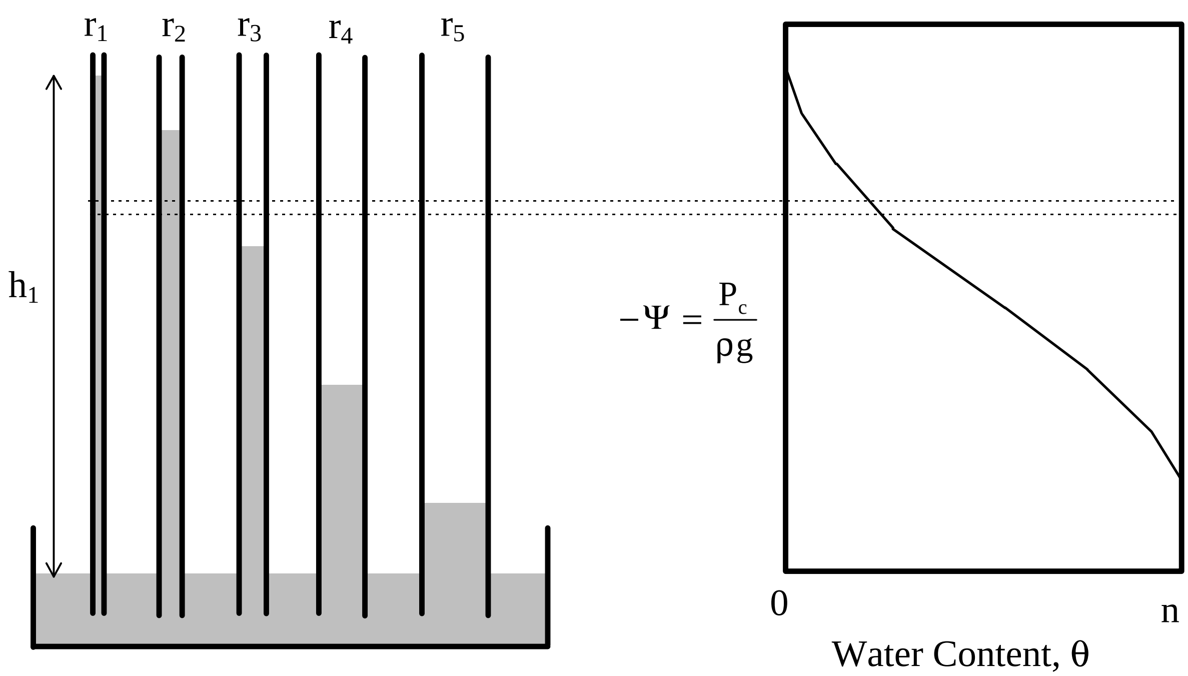
AIR/WATER SYSTEMS AND POROUS MEDIA

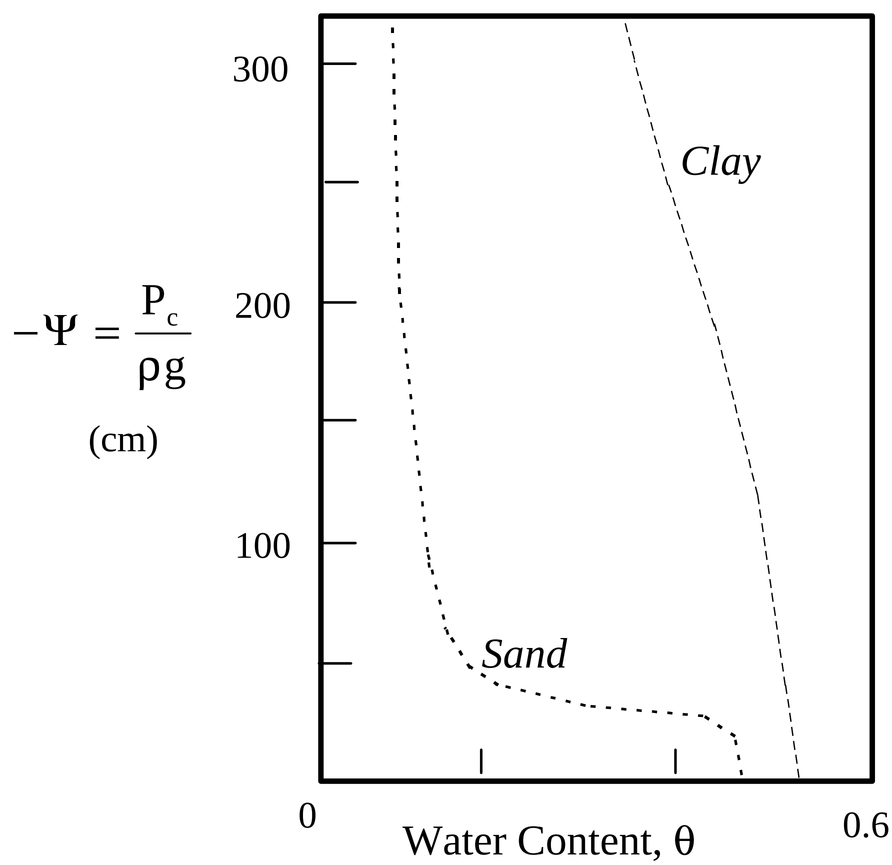
We need a relationship between saturation and capillary pressure to define C(Ψ) [Specific moisture capacity].

Imagine soil as a bundle of capillary tubes:

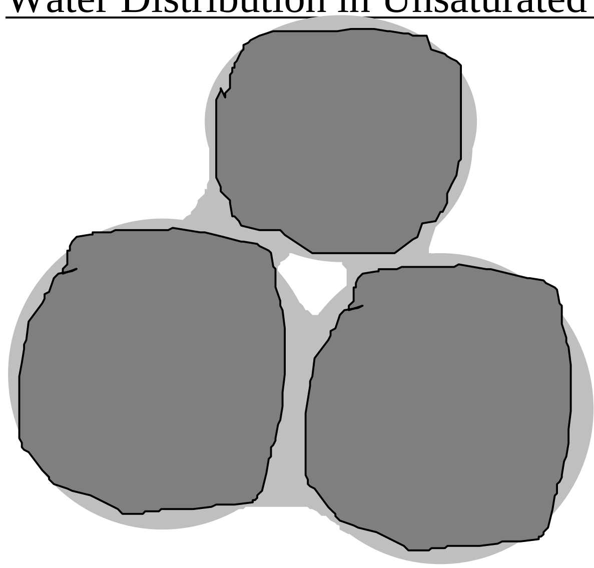
Nonuniform Capillary Model



Soil Water Retention Curves **for different soil types**:



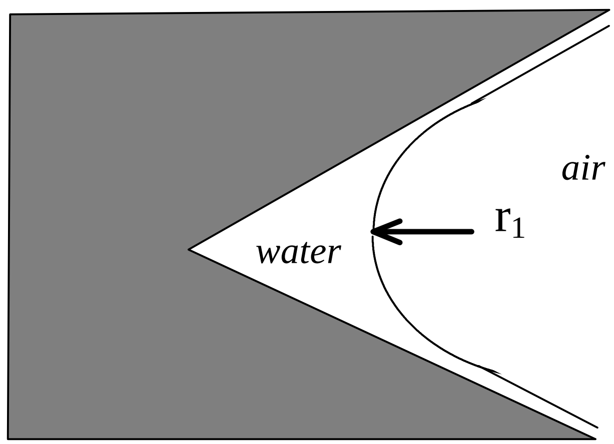
Water Distribution in Unsaturated Soil:

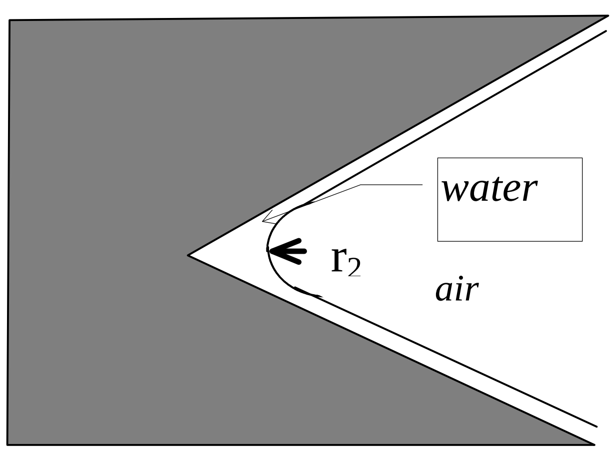


- *Water is wetting fluid-coats solid particles.*
- *Air, nonwetting fluid, exists in larger pore spaces.*
- *Water between soil grains often forms "pendular rings:"*
  - *One radius of curvature in wetting phase.*
  - *Second radius of curvature in nonwetting phase.*

Water Distribution in Unsaturated Soil (continued):

Visualizing θ(Ψ) relationships with the Wedge Model:





As soil drains, water retreats into the wedge.

Radius of curvature decreases, capillary pressure increases.

Can get very high capillary pressures at low saturations, even though the average pore size may not be small.

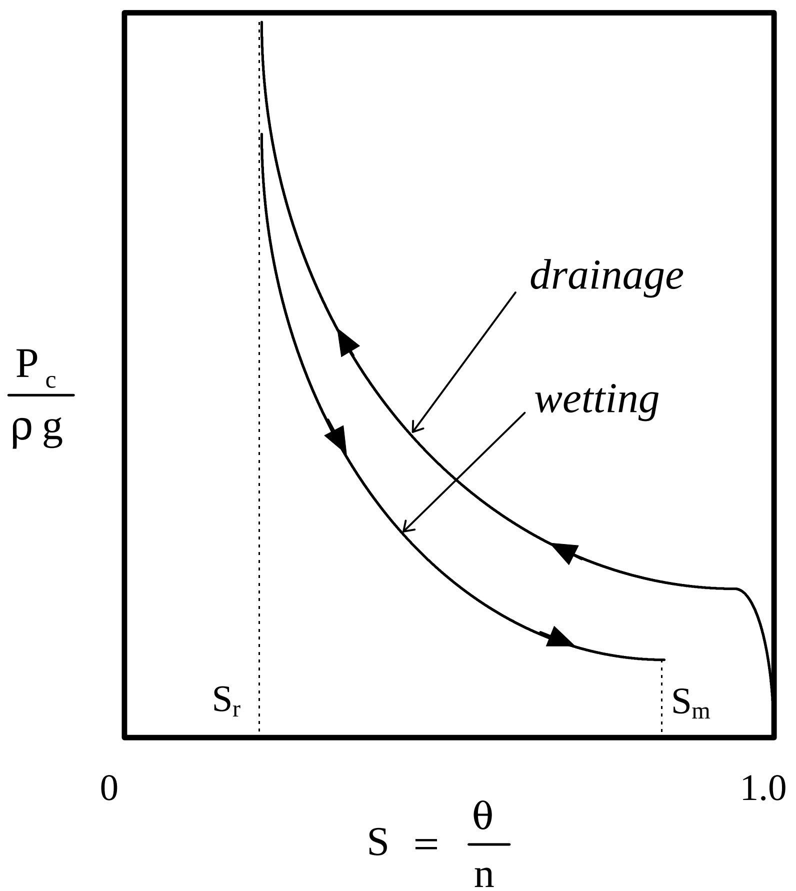
AIR-WATER DISTRIBUTIONS IN POROUS MEDIA:

Capillary Hysteresis

1. Imagine a saturated soil. Drain the soil by increasing air pressure. Plot the capillary head vs. the water content. This is the *drainage* curve.

2. Now allow the soil to become saturated again as the air pressure is slowly decreased. Again plot capillary head vs. the water content. This is the *wetting* or *imbibition* curve.

3. These curves are not the same. Why?

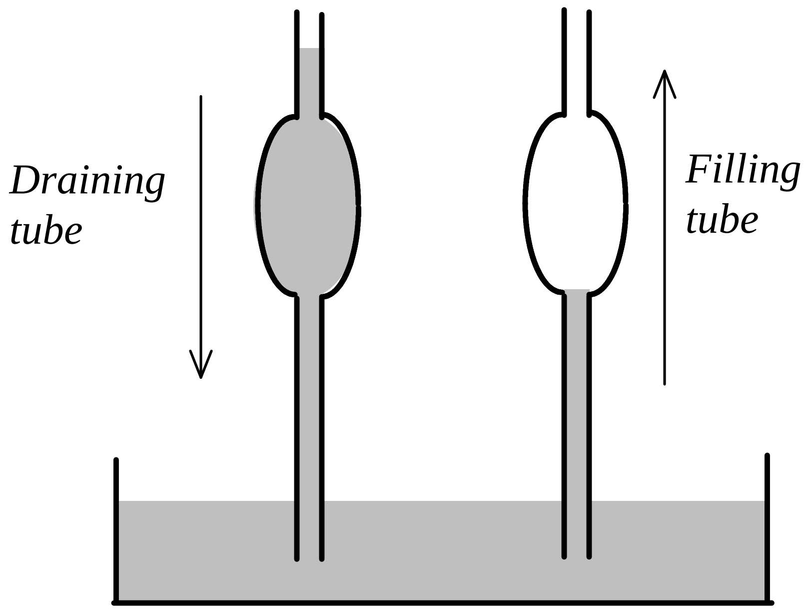


Residual: wetting phase that has drained to an immobile state, S_r.

Entrapped: Discontinuous, non-wetting phase or isolated "ganglia," $1 - S_m$.

AIR-WATER DISTRIBUTIONS IN POROUS MEDIA:

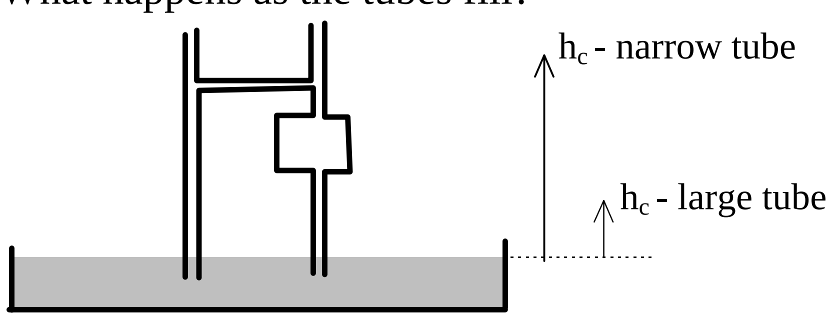
(continued) Capillary Hysteresis:



The draining tube drains to the capillary rise in the narrow part of the tube.

The filling tube cannot rise past the large diameter section because the capillary rise for this size tube is less than its elevation above the pooled water surface.

Imagine capillary tubes of variable diameter that are cross connected. What happens as the tubes fill?

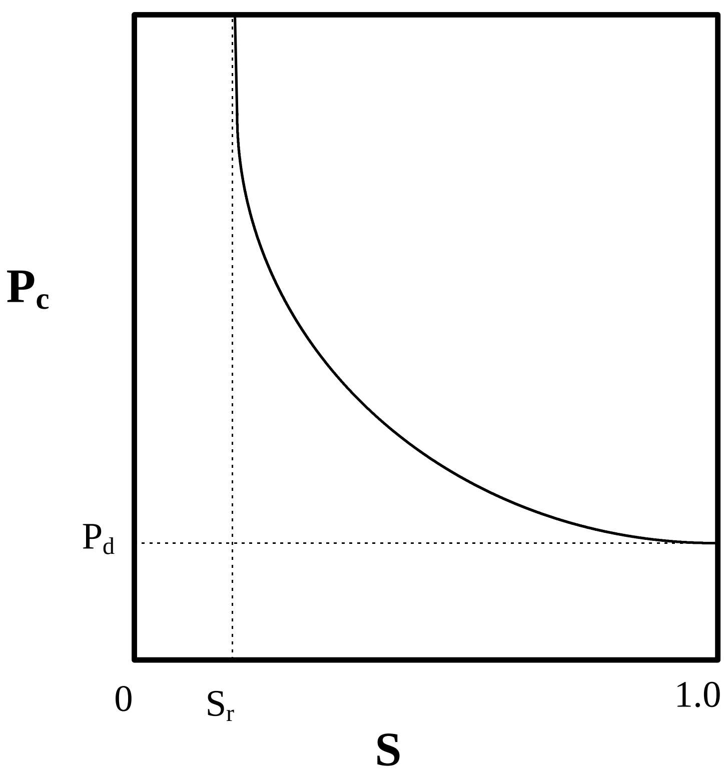


What if there were LNAPL floating on the water surface, and the water table first drops then rises?

Brooks-Corey Model for $S(P_c)$ or $\theta(\Psi)$

Plots of pressure head (P_c or Ψ) versus water saturation (S or θ) are known as:

- capillary pressure-saturation curves (P_c vs. S)
- water retention curves (Ψ vs. θ)
- water characteristic curves (Ψ vs. θ)



Brooks-Corey Model

$$S_e = \left(\frac{P_d}{P_c}\right)^{\lambda}, \; P_c > P_d$$

$$S_e = 1, \; P_c < P_d$$

$$S_e = \frac{S - S_r}{1 - S_r}$$

P_d = Displacement Pressure = P_c at which drainage "first" occurs.
  $P_d/\rho g$ = height of capillary fringe.
S_r = Residual Saturation = S at an arbitarily larger P_c.
  $S_r \approx 0.1 - 0.2$
λ = Pore size distribution index
  $\lambda \approx 2$ average porous media.
  $\lambda \approx 4 - 5$ densely packed/uniformly sized media. (drains ?)
  $\lambda < 1$ well graded and /or structured media. (drains ?)

Brooks-Corey Model for $S(P_c)$ or $\theta(\Psi)$

Remember:

$$\theta = S n$$

$$\Psi = \frac{-P_c}{\rho g}$$

Therefore the Brooks-Corey model can be rewritten:

$$S_e = \frac{\theta - \theta_r}{n - \theta_r} = \left(\frac{\Psi_d}{\Psi}\right)^{\lambda}; |\Psi| > |\Psi_d|$$

$$S_e = 1 \quad (\theta = n); |\Psi| \le |\Psi_d|$$

From this, we can develop a function for specific moisture capacity, $C(\Psi)$:

$$C(\Psi) = \frac{\partial\theta}{\partial\Psi}$$

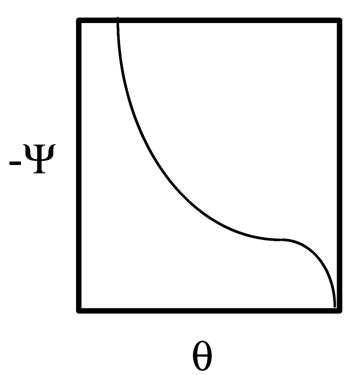
$$C(\Psi) = -\lambda(n - \theta_r)\Psi_d^{\lambda}\Psi^{-\lambda-1}; |\Psi| > |\Psi_d|$$

$$C(\Psi) = 0; |\Psi| \le |\Psi_d|$$

Brooks-Corey model is a discontinuous function. Alternative models exist:

Van Genuchten model:

$$S_e = \frac{\theta - \theta_r}{n - \theta_r} = \left[\frac{1}{1 + (\alpha\Psi)^n}\right]^m$$



Similarity to Brooks-Corey:

$$n = \lambda + 1$$

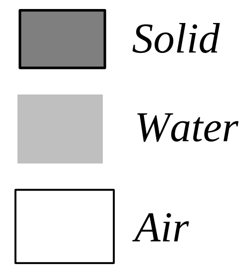
$$m = 1 - \frac{1}{n}$$

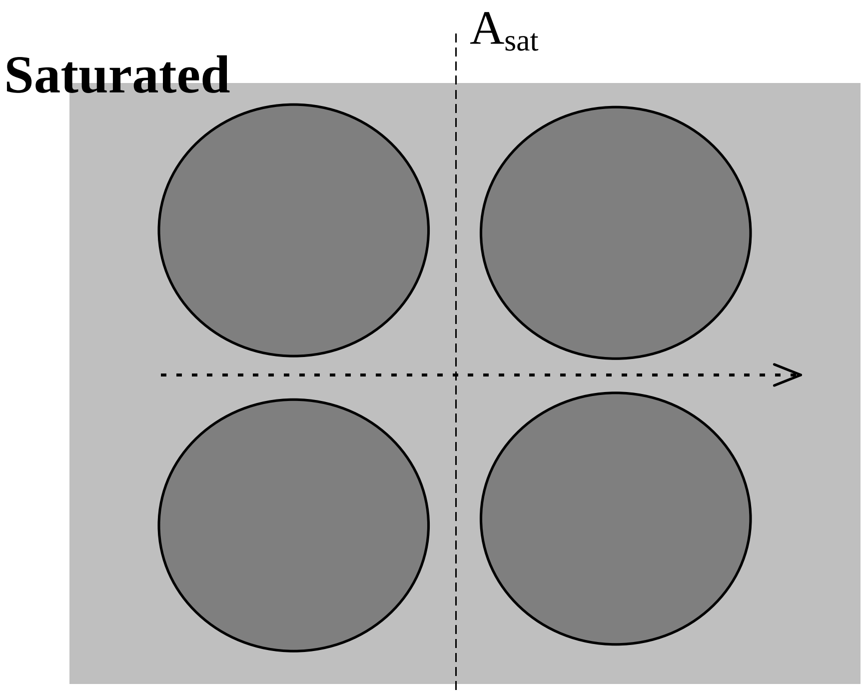
$$\alpha = \Psi_d^{-1}$$

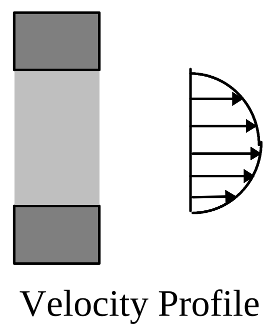
Unsaturated Hydraulic Conductivity, K(θ) or K(Ψ):

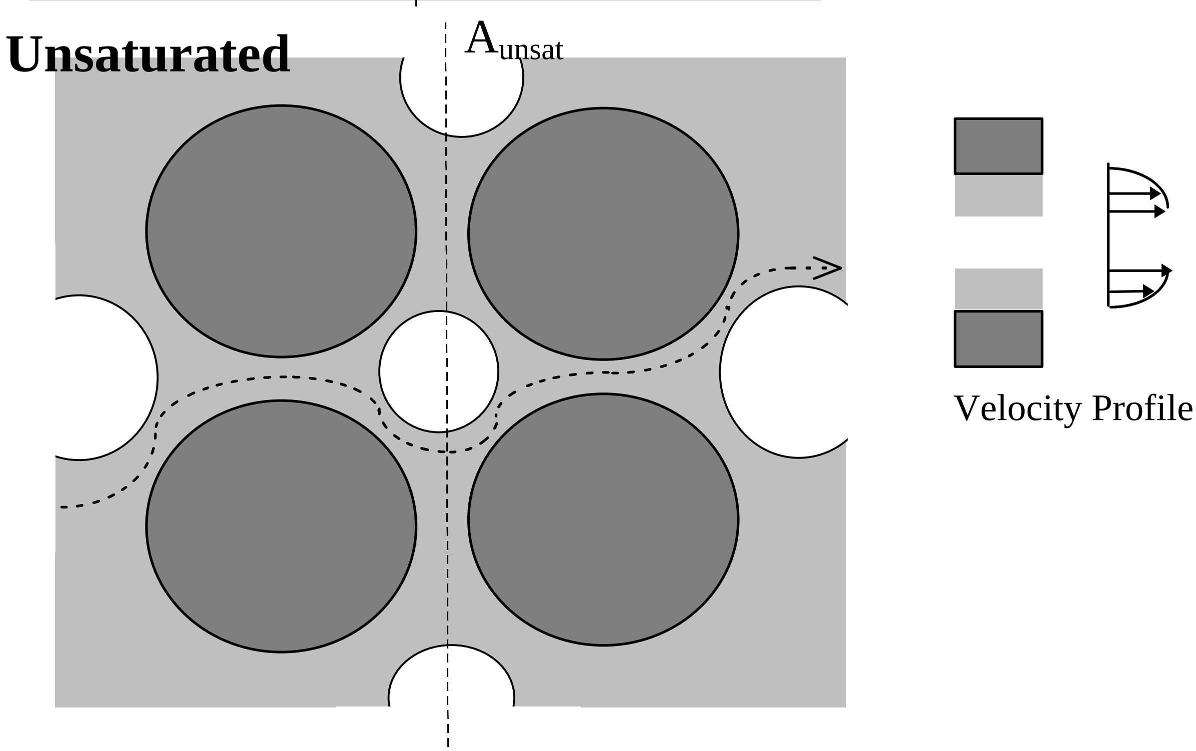
What effect does saturation have on hydraulic conductivity?

1. Hydraulic radius/shear stress at solid/liquid interface.
2. Cross-sectional area available for flow.
3. Tortuosity/path length.









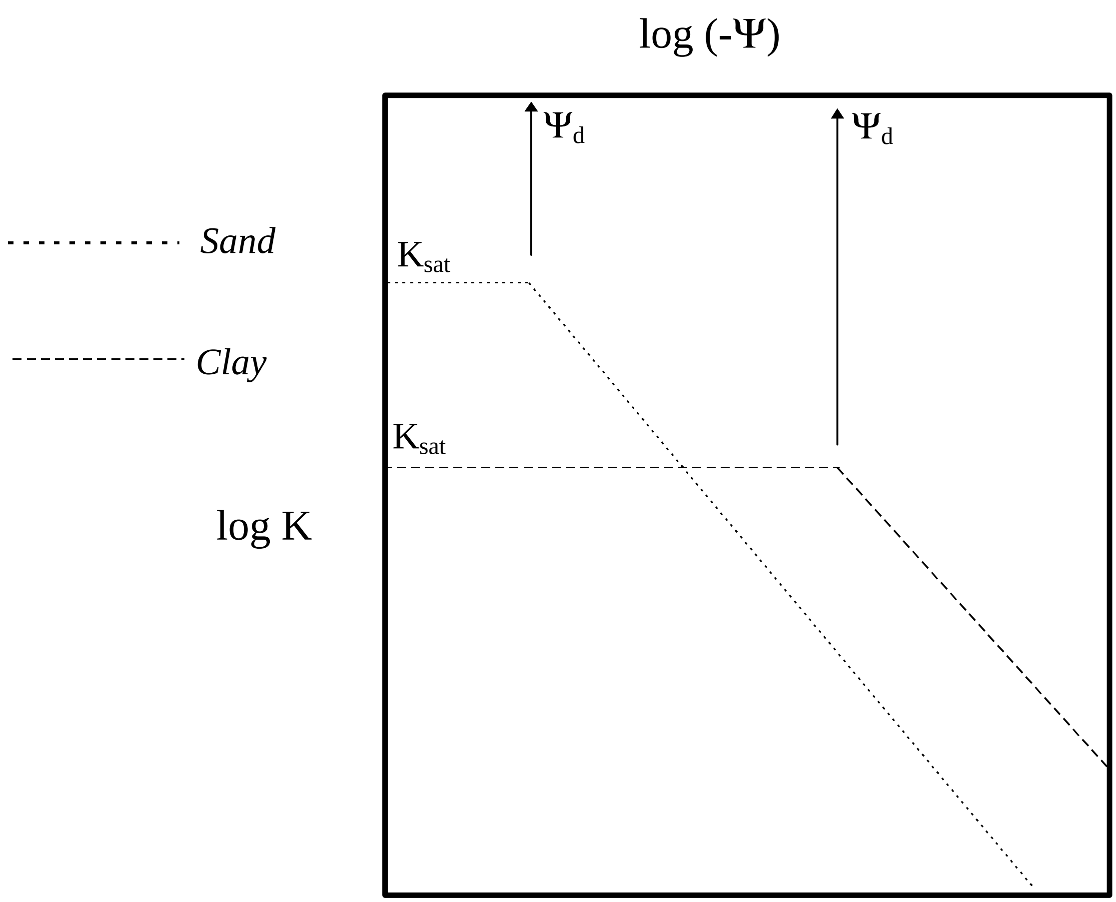
Unsaturated Hydraulic Conductivity, K(θ) or K(Ψ):

$$K(\theta) = K_r K_{sat}$$

K_r = Relative hydraulic conductivity (permeability)
= function of saturation.

$$K_r = S_e^{\frac{2+3\lambda}{\lambda}}$$

$$= \left(\frac{\Psi_d}{\Psi}\right)^{2+3\lambda} \quad ; |\Psi| > |\Psi_d|$$



What is the implication of this relationship on K at high $|\Psi|$?